

Classification of Spine Images using Hybrid Quantum Neural Network Classifier

Ahmed A. Hussein, Ahmed M. Montaser

Electrical Technology Department, Faculty of Technology & Education, Sohag University
Sohag, Egypt
ahmed.ayman@techedu.sohag.edu.eg, Ahmed.Montaser1@techedu.sohag.edu.eg

Hend A. Elsayed*

Electrical Engineering Department, Faculty of Engineering, Damanshour University
Damanshour, Egypt
hend.alielsayed@yahoo.com, Hend.Elsayed@eng.dmu.edu.eg

*Corresponding author: Hend A. Elsayed

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ABSTRACT. *This study classified images of the spine using three different models: the classical neural network, the quantum neural network, and the hybrid quantum neural network with and without principal component analysis. The models were trained on an image size of 32*32 pixels, with similar parameters in terms of range, and the Adam optimizer was used with a learning rate of 0.001. The training took place for 10 epochs, then for 20 and 30 epochs, where a single qubit was used in the quantum and hybrid neural network, which is a small number for this size, which demonstrated the power of quantum neural networks to deal with images of large sizes with the restrictions of using a single qubit, as it showed an accuracy exceeding 97% after many attempts because it depends on a random value for theta. The use of principal components analysis with models has also been strengthened in improving accuracy in neural networks and reducing training time. The successful application of using a single qubit in achieving high accuracy makes quantum neural networks an effective tool. in image classification, which highlights the low computational resources required for quantum computing.*

Keywords: Image classification; Neural network (NN); Quantum neural network (QNN); Hybrid quantum neural network (HQNN); Spine image classification; Principal component analysis (PCA)

1. Introduction. The process of classifying images is intricate and subject to several influences [1]. Within the field of computer vision, image classification is a well-known machine-learning problem. To achieve state-of-the-art performance, various methodologies and techniques have been evaluated on the image classification task [2,3]. Classifying medical images is a crucial use of machine learning in healthcare [4].

A new field of study that blends conventional machine learning (ML) and quantum processing is called quantum machine learning (QML). Using quantum phenomena to improve traditional machine learning algorithms' performance is the main goal of QML. Motivated by the remarkable performance of classical neural networks (NNs), there is a plethora of research being done on their quantum counterpart, which is dubbed quantum neural networks (QNNs). The literature still doesn't provide any conclusive proof that QNNs are better than their classical equivalents, despite the considerable interest in the

subject [5]. Combining quantum computing and neural networks has created a new neural computing paradigm known as (QNNs), which are based on the ideas of quantum mechanics [6]. Combining the best features of both classical ANN and quantum computing, a (QNN) comprises a sequence of quantum operations and either quantum or classical weight parameters between neurons [7].

One subclass of variational quantum algorithms is quantum neural networks, which comprise quantum circuits with parameterized gate operations [8,9]. Quantum circuits with parameters PQC are widely used in quantum machine learning and quantum optimization because of their ability to be trained and optimized for tasks. A parameterized quantum circuit (PQC) is essentially a kind of quantum circuit in which some parameters can be changed, like rotation angles, to accomplish different goals or improve circuit performance. Real numbers are used to represent these parameters, while unitary matrices are used to represent the quantum gates [10].

The latest developments in variational quantum circuits (VQC) have opened a new age of immense potential for quantum computing, which for a long time was only acknowledged for its potential [11]. VQC is a significant class of quantum algorithms that encode data using parameterized quantum circuits and update the circuit parameters using a conventional optimizer. They serve as function approximators, mapping the quantum state holding input features to the state containing the labels by a unitary evolution, much like classical neural networks [12].

Designing quantum neural networks for completely quantum learning tasks has become a significant challenge with the impending arrival of quantum technology. Neural networks have been widely successful in research and industry [13]. Understanding both classical neural networks and quantum computing methods is necessary for building a quantum neural network [14] given the current state of QNN research, a more comprehensive examination of the mismatch between unitary dynamics in quantum computing and dissipative dynamics in neural computing is highly desirable. In addition, the existing QNN is limited by the fact that it can only be trained for larger samples at low latitudes, and there is still work to be done on improving prediction accuracy and performance generalization [15].

The quantum bit, or Qubit Unlike classical bits, which can only be either 0 or 1, quantum bits can exist as both 0 and 1. Superposition is the name given to this phenomenon. In wave physics, the phrase "superposition" refers to the interference of two or more waves. According to quantum physics, a particle is in a superposition state when it becomes incredibly small and begins to behave like a wave. This phenomenon is known as wave-particle duality. The distinction between the two, nevertheless, lies in the fact that this wave collapses and turns into a classical particle when the quantum particle is viewed or measured. Physicists have been unable to comprehend the theory and mechanism underlying this phenomenon, known as the quantum measurement problem, until recently [16,17].

Quantum computers have an operating design that is essentially distinct from classical computers. Unlike classical computers, which use binary digits (bits: 0 or 1) as their operating unit, quantum computers use quantum bits (qubits) [18].

Because of their powerful processing powers, (QNNs) have currently seen some success in the picture categorization space. But unfortunately, when QNNs have more layers, the training procedure is more complicated and the accuracy tends to be lower, which leads to a less stable network. To tackle these obstacles, this manuscript suggests Hybrid quantum neural network (HQNN) [19].

To obtain good classification performance in image classification issues, the descriptiveness and discriminative capability of features extracted are essential [20]. Principal

component analysis, or PCA, is regarded as a crucial method for reducing the dimensions of data in a variety of AI/ML applications. Computer vision and image classification are two of the most significant applications. Owing to its advantages and significance in the categorization of images, principal component analysis (PCA) is utilized not only to uncover significant or recurrent characteristics within high-dimensional data sets but also to reduce their dimensionality. Because of this, PCA is among the finest methods for classifying images and producing extremely accurate results [21].

Like its conventional counterpart, the quantum neural network is trained using a dataset. When the target data is compared with the model's predictions, a loss function is calculated. An optimizer, operating on a conventional computer, iteratively runs the quantum circuit and modifies the parameters θ to find the least loss [22].

This paper compares spinal image classification using three models: classical neural network, quantum neural network, and hybrid quantum neural network with and without principal components analysis to find out which is the most accurate. It tests the ability of a single-qubit quantum neural network to classify images of large sizes relatively large. And test the effectiveness of combining quantum computing and classical computing in image classification and the impact of using principal components analysis with these models. These experiments were performed in the Google Colab environment using a CPU.

2. Problem Formulation. In the field of computer vision, image classification is a well-known problem in machine learning. To achieve advanced performance, different methodologies and techniques have been evaluated in the image classification task. In this work, we examined three different computational models: classical neural network (NN), quantum neural network (QNN), and Hybrid Quantum (HQNN) for spinal image classification to obtain the highest accuracy using the least resources. Furthermore, this study seeks to highlight the potential benefits and limitations of applying quantum computing principles to the image classification problem by exploring the capabilities of QNNs and HQNNs using a single qubit to classify Pictures are of large size.

Each experiment was implemented in the Google Colab environment using CPU resources, ensuring repeatability and accessibility of our results. The computing environment includes the following device specifications:

Python Version: 3.10.12

CPU: Intel(R) Xeon(R) CPU @ 2.20GHz

Physical Cores: 1

Total Cores: 2

3. Proposed Methodology. The general description of the Proposed Methodology is shown in figure1 [23].

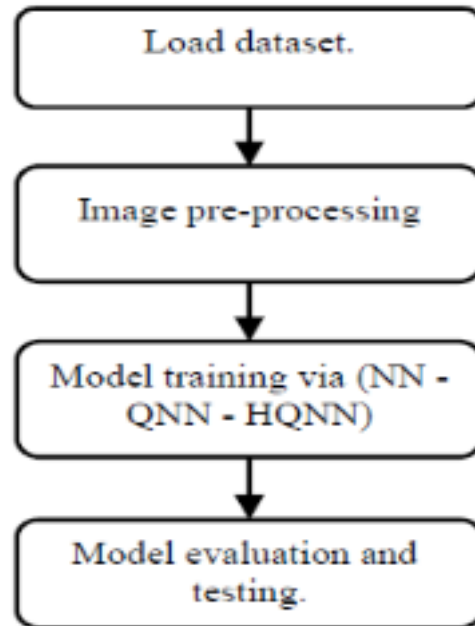


FIGURE 1. Schematic representation of the proposed work

3.1. Data preparation. We started by obtaining a set of vertebral X-ray images from Kaggle to prepare the data. King Abdullah University Hospital served as a site for collecting X-ray images of the vertebrae. We normalized the images into each of the two categories to which each group of images belonged to achieve balance. 80% of the data of each dataset was used for training, and the remaining 20% was used to test the model. The first set of images is the training set. There were 230 images in our collection, of which 112 showed a normal spine and 118 showed a curved spine. The second group is the test group. We used 60 images, including 30 images of the normal spine and 30 images of the curved spine. We pre-processed the image data during the data preparation phase before sending it to the neural network. This involved adjusting the pixel values to ensure they lay between 0 and 1, which improved model convergence and efficiency.

3.2. Image pre-processing. In image processing, picture enhancement is a crucial subject that has seen great success [24]. Figure 2 shows the steps taken for the image pre-processing process, where the images are inserted and resized to the required pixels, such as 32*32 pixels, and they are converted to grayscale images and then normalized Between 0-1 rendering them suitable for seamless processing by the PyTorch framework and the quantum simulator, Principal component analysis (PCA) was then applied to further compress the images.

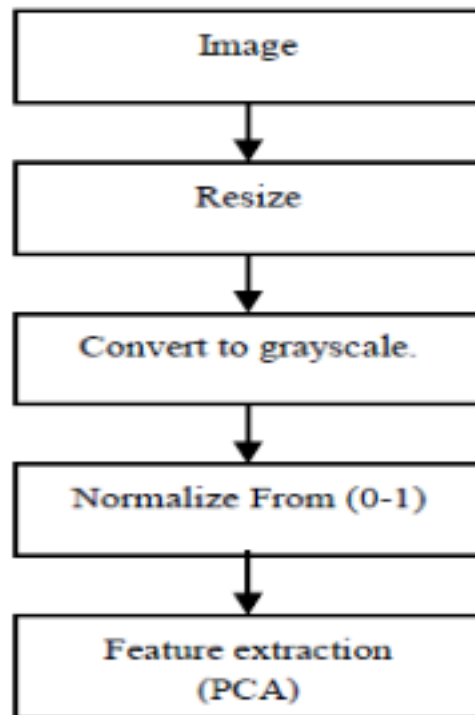


FIGURE 2. Schematic representation of Image pre-processing

3.3. Building a classical neural network (NN). Before the images are fed to the network, the images are pre-processed as mentioned before, such as converting the images to grayscale, shrinking them to a fixed size, and normalizing them, which makes them suitable for seamless processing and feature extraction. The model was built using Kera's framework and TensorFlow, where the network consists of:

(1) The input and flattening layer, where the construction of the network begins with a flat layer, which converts the input images from a two-dimensional matrix to a one-dimensional matrix, and this process prepares the spatial image data for processing through dense layers.

(2) (Dense layer 1) The ReLU activation function is used. The choice of ReLU is due to its effectiveness in introducing non-linear properties to the network, allowing it to learn complex patterns by passing only positive values and nullifying negative values.

(3) (Dense layer 2) acts as the output layer and contains several neurons equal to the classification categories. The softmax activation function is applied here to transform the linear output of the class into a probability distribution over the respective classes. The model was assembled with the following specifications to prepare it for the training phase. The binary loss and entropy function was chosen due to its suitability for binary classification problems. Optimizer the Adam optimizer was used with a learning rate of 0.001, the batch size was set to 1, and training was conducted over a period of 10 epochs and then for 20 epochs. Accuracy was chosen as a typical performance to measure performance. Therefore, the general design of the neural network (flat layer, dense layers, output layer) is as it is shown in Figure 3.

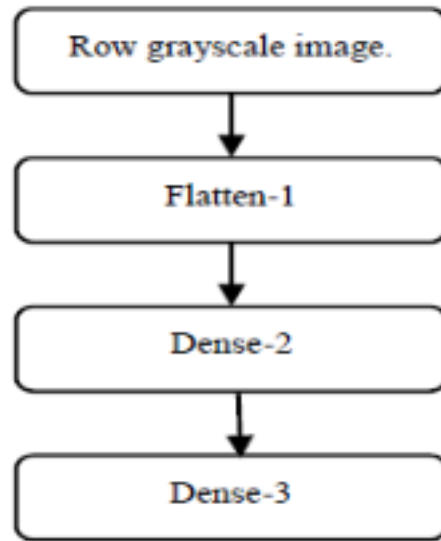


FIGURE 3. Structure of a neural network (NN)

3.4. Building a quantum neural network (QNN). After preprocessing, flattening, and converting images into one-dimensional matrices, a new quantum neural network architecture is presented to leverage a simplified quantum circuit for the image classification task. This method uses 1 qubit to encode and process image data.

A fundamental quantum circuit with Hadamard gates was used to generate superposition states. And RY spin gates, whose parameters are determined by inputting flat image data and parameters (X and Theta parameters), where classical data is encoded into quantum states, using quantum mechanics to process and classify images, and the quantum circuit ends with the process of measuring and translating quantum states into classical outputs, which is used to predict the image category. Based on the probability distribution obtained from quantum measurements. The quantum circuit concludes with a measurement process that transforms quantum states into classical outputs necessary for predicting image classification. Using probability distributions derived from quantum measurements, a quantum neural network undergoes recursive training. During training iterations, the rotation gate (RY) parameters are optimized to reduce classification errors. Training involves performing forward passes, during which the output probabilities for each class are calculated and the parameters are updated accordingly. Adjustments to quantum circuit parameters are made using a direct update rule, which is based on differences between expected and actual labels and aims to minimize classification loss.

The quantum neural network was trained for 10 epochs and then for 20 epochs using the QASM simulator and the Adam optimizer, with a learning rate of 0.001 and a batch size of 1. Figure 4 shows a quantum circuit designed for a single qubit, with the number of blocks determined dynamically. By image size or number of features, expressed.

- $N = (\text{Image size} * \text{Image size (or) number of components})$
- c (control): The number of control qubits involved in a gate operation.
- q (qubit): The number of qubits the circuit operates on.

Control qubits regulate the impact of a gate operation on other qubits inside the circuit. In essence, they function similarly to a switch, with the ability to switch the action on or off based on their status.

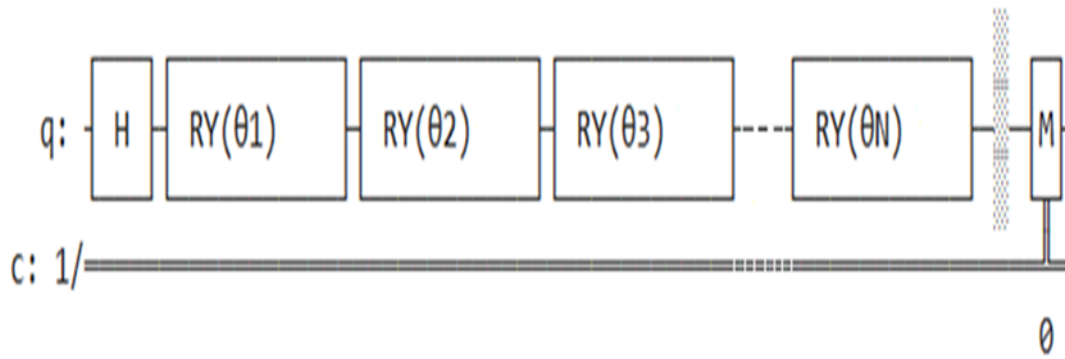


FIGURE 4. Quantum circuit for 1 qubit

3.5. Building a hybrid quantum neural network (HQNN). In this model, there is a new approach to image classification, where the principles of quantum computing are combined with the structure of classical neural networks, which leads to the creation of a hybrid quantum neural network (HQNN).

This model takes advantage of the strengths of both models, such as the strength of the classical neural network in processing high-dimensional features and potential computational advantages. Which is provided by quantum circuits, where the network begins with a series of fully connected layers and converts the input image data into a form that can be processed by the quantum layer. These layers include:

(1) An input layer (flattening) that flattens the image and places it in an area with higher dimensions.

(2) Intermediate layers with Relu activations to enhance the network's ability to learn complex patterns

(3) The pre-quantum layer, which works to condense the information into one-dimensional outputs prepared for quantum processing.

The HQNN is constructed using Qiskit for the quantum elements and PyTorch for classical components, aiming to harness the strengths of both computational approaches in tackling image classification tasks implement a quantum circuit with Hadamard gates to generate superposition states, followed by spin gates (RY), which encode the outputs of the classical layers into qubits. Finally, the measurement process takes place, and the quantum state is transformed into classical outputs that reflect the processed features. The network was trained to the same data set using Qiskit Aer simulator, using only 1 qubit, and an Adam optimizer with a learning rate of 0.001, The batch size is set to 1, Figure 5 shows the layers of the hybrid quantum neural network structure Where it means:

- Flatten-1 :Takes the image data and flattens it into a 1D vector.
- Linear-2:Fully-connected layer that performs a linear transformation on the flattened data.
- Linear-3:Another fully-connected layer that further processes the data from the previous layer.
- Linear-4: the final fully-connected layer in the classical part, generating features for the quantum circuit.
- Hybrid:This layer integrates the classical neural network with the quantum neural network The circuit in the hybrid layer is the 1-qubit quantum circuit that was used.

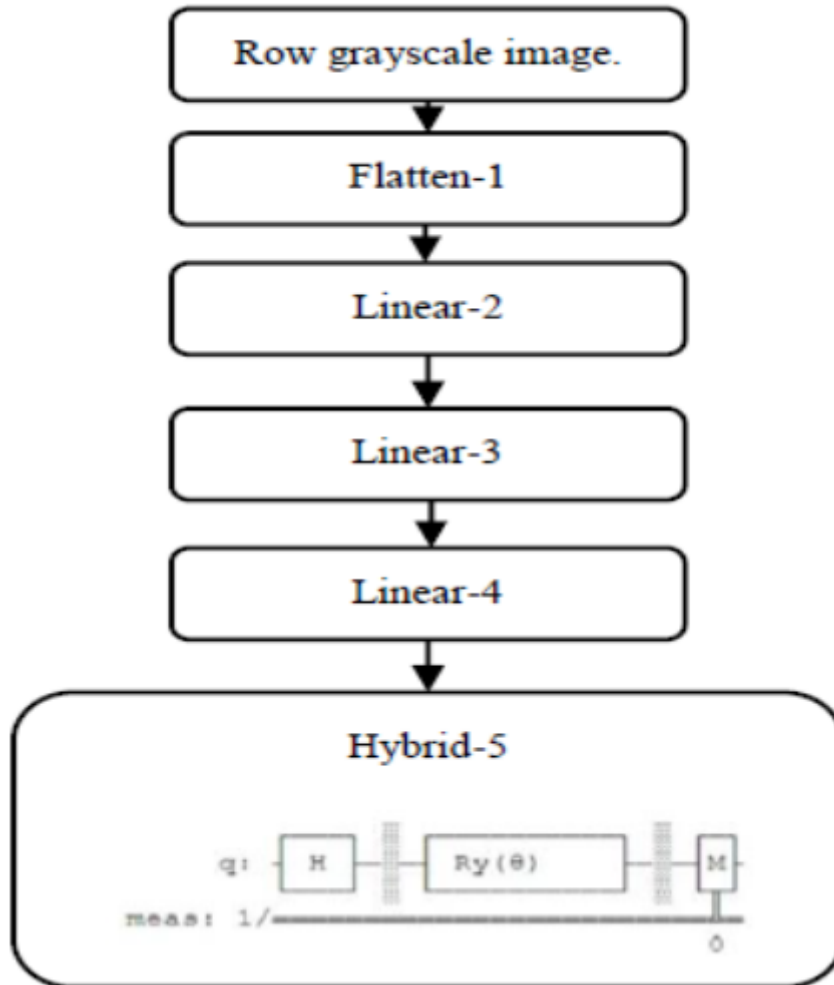


FIGURE 5. Structure of hybrid quantum neural network (HQNN)

4. Results and Discussion.

4.1. Model results after 10 epochs. This study focused on comparing the performance of classical neural network (NN), quantum neural network (QNN), and hybrid quantum neural network (HQNN) models with and without applying principal component analysis (PCA) in image classification, where all models were trained. With a single batch size and a number of parameters close in range. A single qubit was used in quantum and hybrid models, making use of a randomly generated theta parameter, which was optimized through a learning and training process. All models were tested on a baseline dataset of 32*32 pixels, which is a large size considering the context of quantum computing. A feature number of 100 was used for all models with PCA. The training was conducted for 10 training periods (epoch) as shown in Table 1, where the results showed that:

1- With PCA, the accuracy of the classic neural network test improved from 70% to 93%, and the time decreased from 24 seconds to 13 seconds, which indicates the ability of PCA to enhance the efficiency of the model.

2- The quantum neural network model that works with a single qubit achieved a test accuracy of 53%, with a PCA of 57%, and a decrease in time from 300 seconds to 75 seconds, because it relied on a random value of theta.

3- With PCA, the accuracy of the hybrid quantum neural network test also improved from 85% to 95%, and the time decreased from 174 seconds to 137 seconds. This indicates that it is the highest model in terms of accuracy over 10 epochs, and this indicates the effectiveness of integrating and benefiting from quantum computing with classical networks.

TABLE 1. Model results after 10 epochs.

Model	Batch size	Time (sec)	Training Accuracy%	Test Accuracy%	learning rate
NN	1	24	81	70	0.001
NN PCA	1	13	100	93	0.001
QNN	1	300	50	53	0.001
QNN PCA	1	75	50	57	0.001
HQNN	1	174	87	85	0.001
HQNN PCA	1	137	100	95	0.001

Figures 6, 7, 8, 9, 10, 11, and 12 show the accuracy and loss curves for the classical neural network model, the quantum neural network, and the hybrid quantum neural network with and without principal components analysis after 10 epochs.

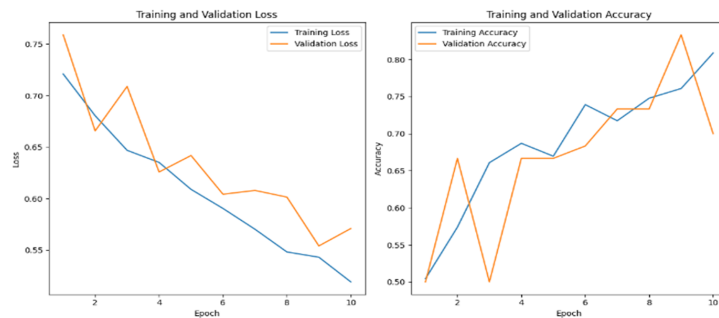


FIGURE 6. Loss function and accuracy of training and testing data for the NN model for 10 epochs.

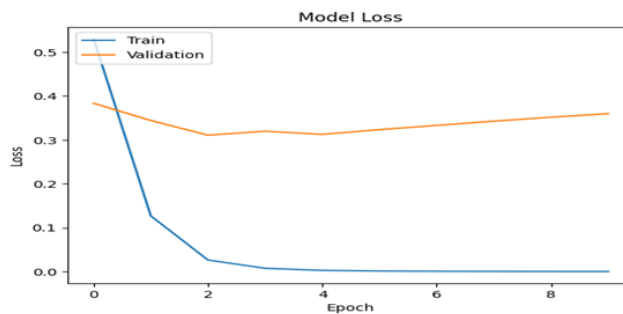


FIGURE 7. Loss function of training and testing data for NN (PCA) model for 10 epochs.

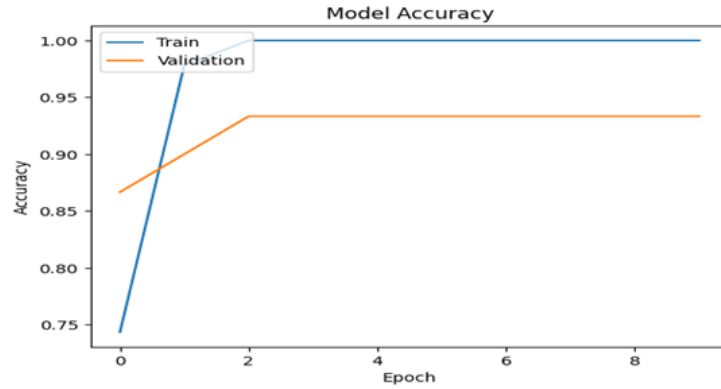


FIGURE 8. Accuracy function of training and testing data for the NN (PCA) model for 10 epochs.

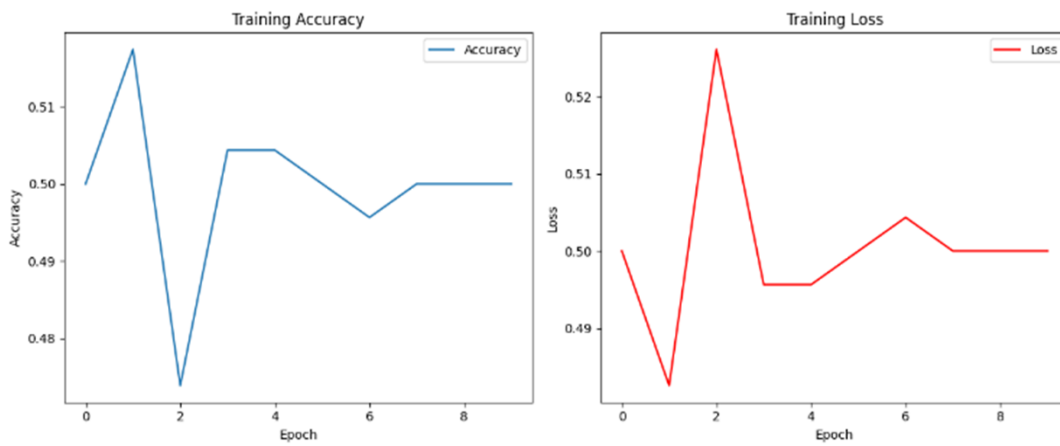


FIGURE 9. Loss function and accuracy of training and testing data for the QNN model for 10 epochs.

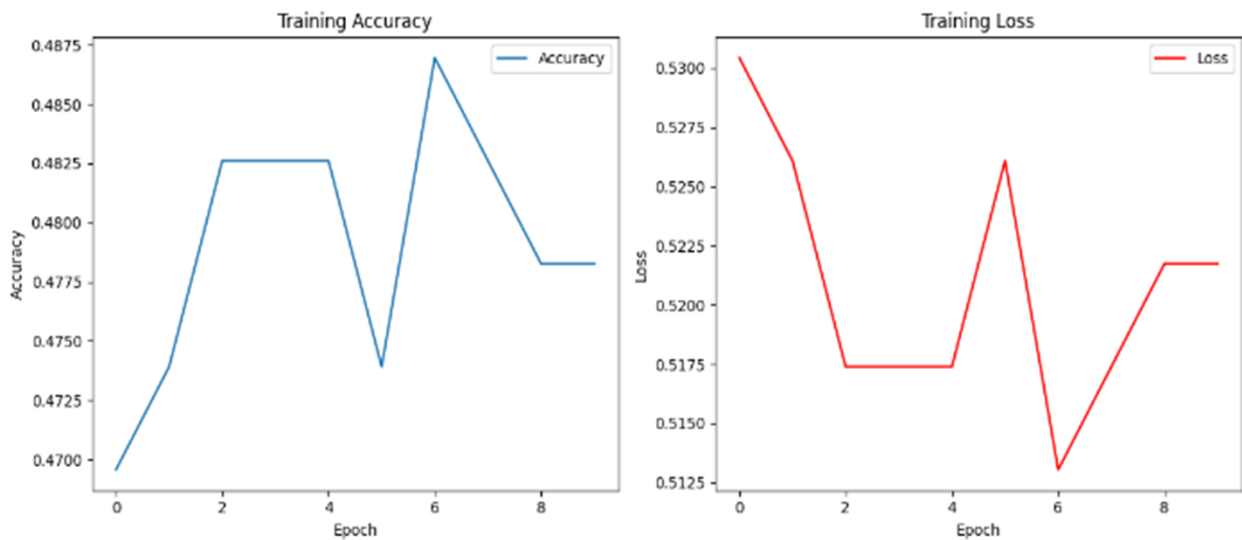


FIGURE 10. Loss function and accuracy of training and testing data for the QNN (PCA) model for 10 epochs.

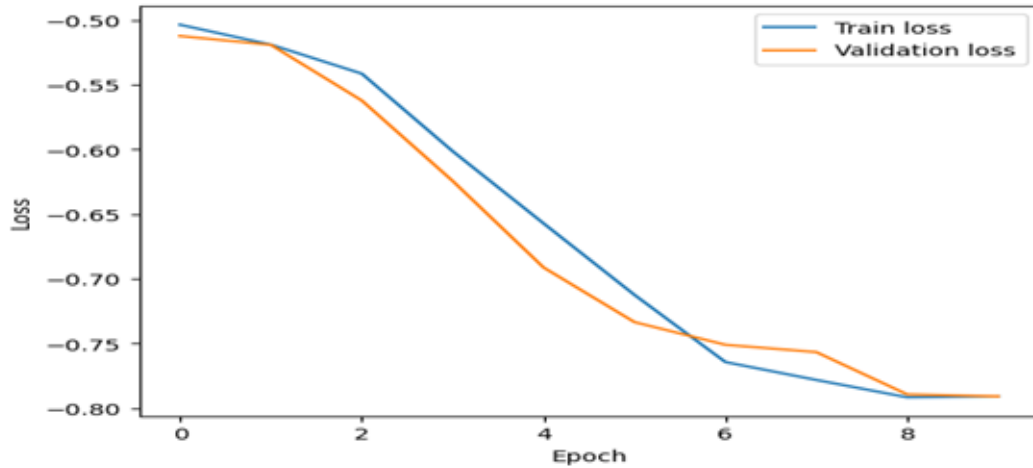


FIGURE 11. Loss function of training and testing data for HQNN model for 10 epochs

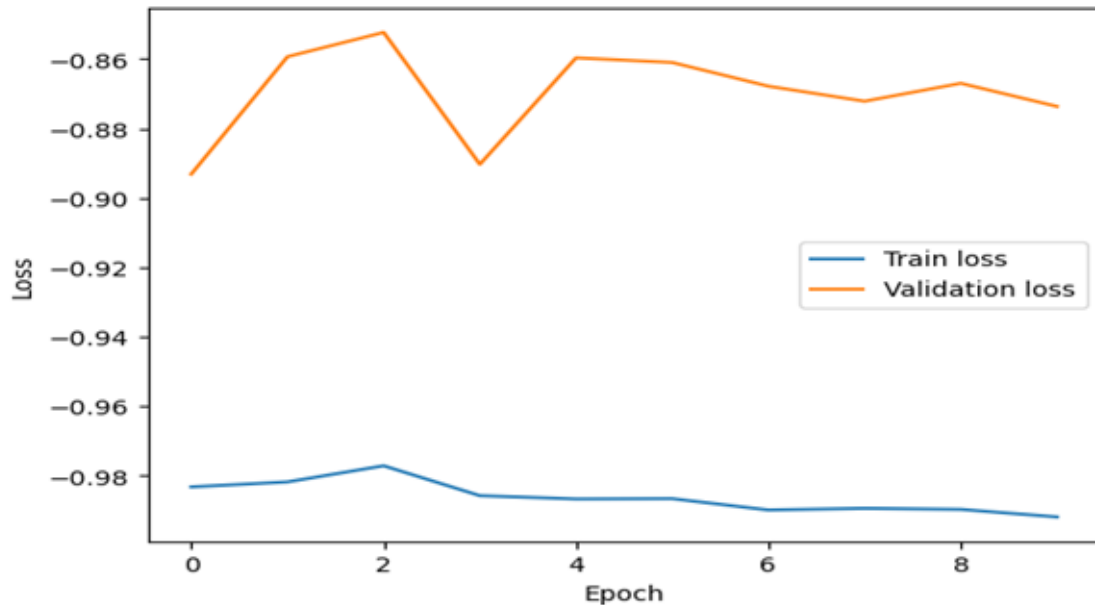


FIGURE 12. Loss function of training and testing data for HQNN (PCA) model for 10 epochs

4.2. Model results after 20 epochs. We note that the accuracy of all models increased with the increase in the number of iterations, especially the noticeable increase in quantum neural networks, whose accuracy increased from 53% to 97% after many attempts because it depends on the random value of theta. Therefore, it achieves the highest accuracy of all models, and this is considered a major achievement, as a single qubit was used to manage a data set of $32 * 32$ pixels, which confirms the ability of quantum models to process high-dimensional data, and this is a promising direction for quantum computing Using a single qubit for image classification tasks, especially at relatively large sizes, has proven to be more effective than expected. Quantum models have handled the complex feature space of high-resolution images using a single qubit, a strong testament to the potential of quantum computing for future developments in quantum computing. The integration of principal components analysis (PCA) with quantum models also led to shedding light

on the process of balancing between reducing features and retaining information. The use of PCA with models also led to enhancing accuracy and reducing time. Table 2 shows the model results after 20 epochs.

TABLE 2. Model results after 20 epochs

Model	Batch size	Time (sec)	Training Accuracy%	Test Accuracy%	learning rate
NN	1	32	92	87	0.001
NN PCA	1	11	100	95	0.001
QNN	1	594	98	97	0.001
QNN PCA	1	132	48	57	0.001
HQNN	1	323	92	88	0.001
HQNN PCA	1	261	100	93	0.001

Figures 13, 14, 15, 16, 17, 18, and 19 show the accuracy and loss curves for the classical neural network model, the quantum neural network, and the hybrid quantum neural network with and without principal components analysis after 20 epochs.

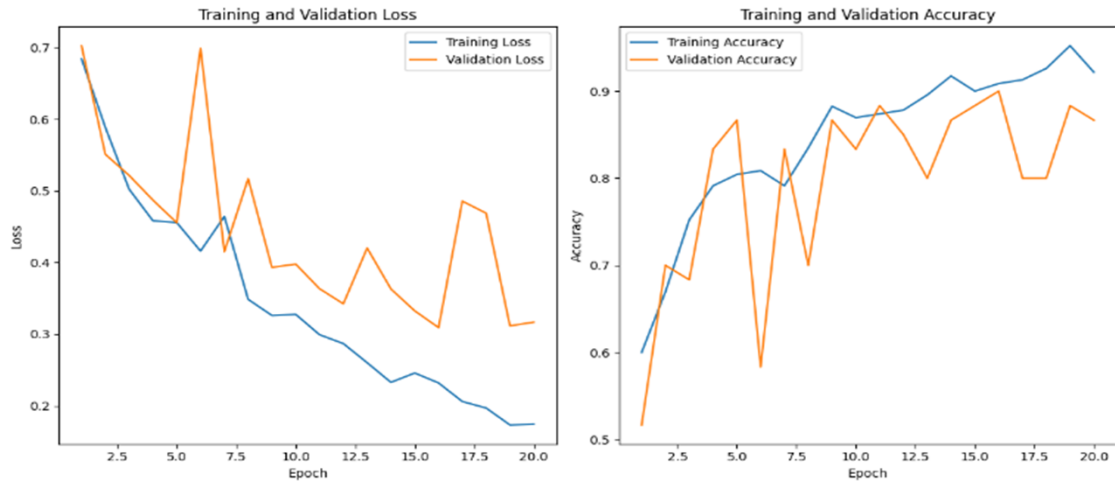


FIGURE 13. Loss function and accuracy of training and testing data for the NN model for 20 epochs.

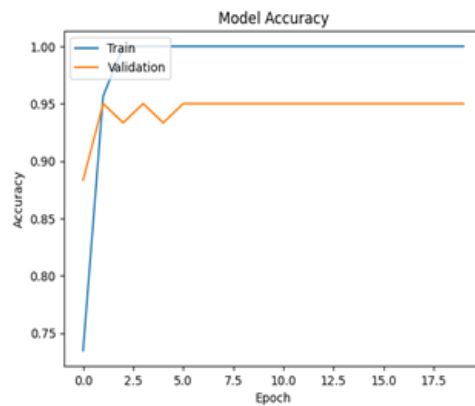


FIGURE 14. Accuracy function of training and testing data for the NN (PCA) model for 20 epochs

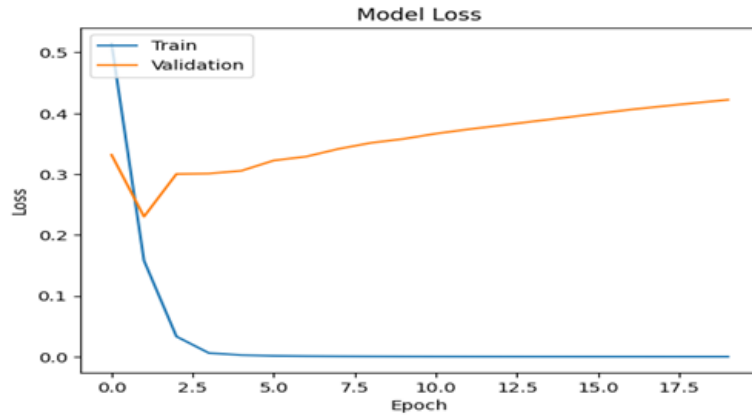


FIGURE 15. Loss function of training and testing data for NN (PCA) model for 20 epochs.

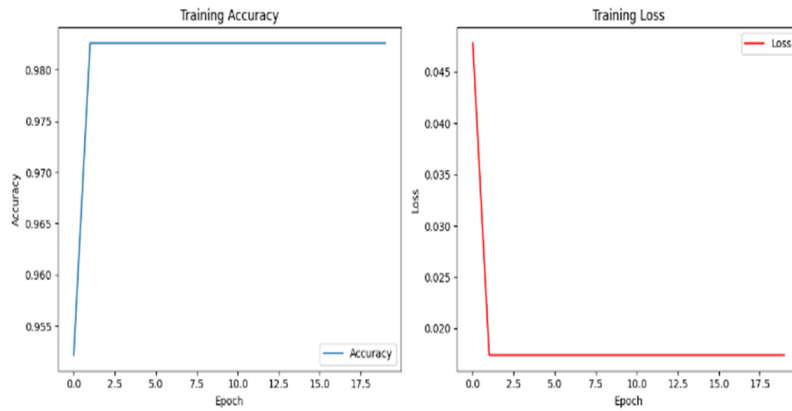


FIGURE 16. Loss function and accuracy of training and testing data for the QNN model for 20 epochs



FIGURE 17. Loss function and accuracy of training and testing data for the QNN (PCA) model for 20 epochs.

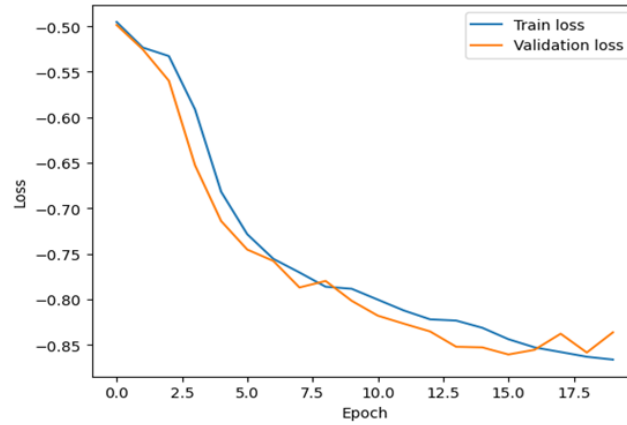


FIGURE 18. Loss function of training and testing data for HQNN model for 20 epochs

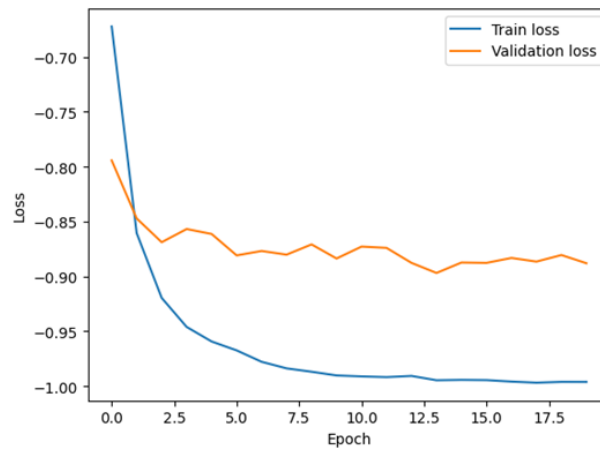


FIGURE 19. Loss function of training and testing data for HQNN (PCA) model for 20 epochs

4.3. **Model results after 30 epochs.** Training was conducted for 30 epochs following this work [25]. Table 3 shows the results of the models after 30 epochs.

TABLE 3. Model results after 30 epochs

Model	Batch size	Time (sec)	Training Accuracy%	Test Accuracy%	learning rate
NN	1	81	96	90	0.001
NN PCA	1	22	100	92	0.001
QNN	1	564	53	45	0.001
QNN PCA	1	225	50	45	0.001
HQNN	1	515	87	90	0.001
HQNN PCA	1	451	100	92	0.001

5. **Conclusion.** In conclusion, in this paper, we compared the classical neural network with the quantum neural network and the hybrid quantum neural network with and without principal component analysis for classifying spinal images. The results of this study confirmed the capabilities of quantum computing in image classification, as the use

of a single qubit in quantum and hybrid neural networks proved its effectiveness. In dealing with relatively large sizes, a high accuracy was obtained for the quantum neural network model with 1 qubit, exceeding 97% this after several attempts, and this is because it depends on a random value of theta that is updated. The hybrid quantum neural network model also showed better accuracy than the classical neural networks, which indicates the success of integrating quantum computing with classical ones. The use of principal components analysis with models also led to enhancing performance while preserving information and reducing training time. It was also noted that the accuracy depends on the number of epochs. This sets a standard for future investigations into quantum computing applications and demonstrates the fundamental benefit. For quantum models to navigate complex feature spaces. Our results go beyond the limits of conventional computing and herald a new era of data processing and analysis capabilities emerging from quantum neural networks, especially the elegant, powerful, and direct one-qubit QNN.

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